A NUMERICAL SOLUTION FOR LATERAL AND TORSIONAL BUCKLING OF BEAMS AND COLUMNS

BY

CHARLES PAO-CHUN TUNG

B.S., St. John's University, 1945
M.S., University of Illinois, 1948

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SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
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Introduction

Structural members of thin-walled open section (which includes I-sections, Z-sections and channel sections) have been observed to buckle by twisting, as well as by bending, because of their relatively weak strength in torsion. This buckling phenomenon is called lateral buckling in beams which are subjected to transverse loads only, torsional buckling in columns which are subjected to longitudinal loads only, and lateral-torsional buckling in beam-columns which are subjected to both transverse and longitudinal loads. Mathematically, the nature of the three kinds of buckling is traceable to one common origin. However, for the sake of clearer physical description, differentiation into three kinds is desirable. The first two terms, lateral and torsional buckling, are of well-established usage in previous literature. The third term, lateral-torsional buckling the nature of which obviously comprehends those of the first two, corresponds to the usual term of torsional-flexural buckling.

Although the theoretical background of lateral, torsional, and lateral-torsional buckling has been well established, there still remain a great many important problems of practical nature which, owing to their difficult boundary conditions and the intricacy of their resultant mathematical expressions, have never been satisfactorily solved. With the rapid progress of modern engineering science, the requirements of weight-saving necessitate the wide use of thin-walled open sections. It becomes of the utmost importance that a general method of analysis be made available to solve the difficult problems in lateral, torsional and lateral-torsional buckling as well as the simple ones, the solutions of which are already well known in the literature. It is with this objective that the present

investigation is undertaken.

A Numerical Procedure

The method presented herein for the solution of lateral and torsional buckling problems in beams and columns is a numerical procedure which in turn is a combination of two well-known procedures. A method of successive approximations is used in conjunction with the numerical procedure of integration published by N. M. Newmark.[1] In essence, the process is as follows: first a deformation function is assumed, and then a new deformation function is determined by numerical integration of the differential equation. This process is repeated until the assumed and computed deformation functions agree. The solutions usually converge rapidly for all problems except a very few which will be treated as special cases.

The numerical solution is approximate but can be made as accurate as is desired by taking a sufficient number of divisions along the span of the member. As the number is increased, the accuracy of the solution increases. For most practical problems no more than eight to twelve divisions are needed. The numerical computations are straightforward and require only an elementary knowledge of mathematics. For most problems slide rule accuracy is sufficient.

Illustrative Examples

The numerical method presented is applicable to the solution of lateral and torsional buckling problems in beams and columns with different boundary conditions, various loading conditions, cross-sectional variations along the span, and different structural shapes. The procedure is explained and illustrated through the use of examples which are divided into two groups, illustrative examples in I-sections, and illustrative examples in other than I-sections including Z-sections and channel sections. These examples which are twenty-four in

1. "Numerical Procedure for Computing Deflections, Moments, and Buckling Loads," by N. M. Newmark, Transactions, ASCE, Vol. 108, 1943, p. 1161.

number are designed to represent the most typical cases from which the numerical solutions for more complex problems can be readily derived.

Design Recommendations

The current methods of design of structural members against lateral, torsional, and lateral-torsional buckling, as provided in the specifications, are critically examined. From the study of the lateral and torsional buckling problems solved, design recommendations for I-sections which are based on a fundamental consideration of the limiting stability of the member are proposed. The concept of "safe stress curve" for the basis of design is introduced.

Concluding Remarks

While the primary aim of the present study is directed toward the investigation of members of structural steel, the results obtained herein can also be used to advantage to investigate structural members in aeronautical engineering and other related branches of engineering. Furthermore, the general procedure described is applicable to many other problems, since it permits a relatively simple and accurate numerical integration of a class of differential equations. The numerical solution is believed to be a possible approach in solving problems in lateral and torsional buckling of members stressed beyond the proportional limit. However, the subject is beyond the scope of this work.

VITA

Charles Pao-Chun Tung was born on May 25, 1925, in Haimen, Kiangsu, China. After graduating from St. John's Middle School in 1943, he entered St. John's University, Shanghai, China and received from that institution the degree of Bachelor of Science in Civil Engineering in September 1945. From October 1945 to July 1947 he was a cadastral surveyor for the Bureau of Land Administration of the Shanghai Municipal Government. In the United States he was appointed Engineer Aide I in the Bureau of County Roads and City Streets of the Illinois State Highways, Springfield, Illinois, in the summer of 1948. In the summer of 1950 he worked for Strobel and Salzman, Consulting Engineers in New York, N. Y., in designing bridges, buildings, and various other steel and reinforced concrete structures.

He began his graduate study at the University of Illinois in September 1947 and received the degree of Master of Science in June 1948. In February 1949 he was appointed Research Assistant in the Structural Research Division in the Civil Engineering Department at the University, which position he now holds.

He was elected an associate member of Sigma Xi in November 1949, and a member of Phi Kappa Epsilon in May 1950.

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